LA TRASFORMATA DI FOURIER

1 PROPRIETA' FONDAMENTALI

1) Trasformazione diretta ed inversa

$$G(f) = \mathbf{F}[g(t)] = \int_{-\infty}^{+\infty} g(t)\, e^{-j2\pi f t}\, dt \;; \qquad g(t) = \mathbf{F}^{-1}[G(f)] = \int_{-\infty}^{+\infty} G(f)\, e^{j2\pi f t}\, df$$

$$g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f)$$

2) Rappresentazione complessa

$$G(f) = \mathbf{Re}\{G(f)\} + j\,\mathbf{Im}\{G(f)\} = |G(f)|\exp[j\Phi_G(f)];$$

$$|G(f)| = \sqrt{\left(\mathbf{Re}\{G(f)\}\right)^2 + \left(\mathbf{Im}\{G(f)\}\right)^2}\;; \qquad \Phi_G(f) = \operatorname{arctg}\left[\mathbf{Im}\{G(f)\} \big/ \mathbf{Re}\{G(f)\}\right].$$

3) Valore nell'origine

$$G(f=0) = \int_{-\infty}^{+\infty} g(t)\, dt \;; \qquad g(t=0) = \int_{-\infty}^{+\infty} G(f) \cdot$$

4) Linearita'

$$c_1 g_1(t) + c_2 g_2(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} c_1 G_1(f) + c_2 G_2(f) \cdot$$

5) Simmetria

$$g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) \Leftrightarrow G(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} g(-f) \cdot$$

6) Inversione dell'asse

$$g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) \Leftrightarrow g(-t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(-f) \cdot$$

7) Coniugazione

$$g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) \Leftrightarrow g^*(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G^*(-f) \;\text{ ed anche }\; g^*(-t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G^*(f) \cdot$$

8) Proprieta' di simmetria

Se g(t) e' un segnale reale, allora risulta:

$$\mathbf{F}[g(t)] = G(f) = G^*(-f) \Leftrightarrow \begin{cases} \mathbf{Re}\{G(f)\} = \mathbf{Re}\{G(-f)\};\; \mathbf{Im}\{G(f)\} = -\mathbf{Im}\{G(-f)\}; \\ |G(f)| = |G(-f)|; \qquad \Phi_G(f) = -\Phi_G(-f). \end{cases}$$

Se g(t) e' un segnale reale e pari, anche G(f) e' reale e pari:

$$G(f) = \mathbf{Re}\{G(f)\} = G(-f) \quad ; \quad \mathbf{Im}\{G(f)\} = 0 \,;$$
$$G(f) = 2\int_{0}^{+\infty} g(t)\cos(2\pi f t)\,dt \,.$$

Se g(t) e' un segnale reale e dispari, allora G(f) e' puramente immaginaria e dispari:

$$G(f) = j\,\mathbf{Im}\{G(f)\} = -G(-f) \quad ; \quad \mathbf{Re}\{G(f)\} = 0 \,;$$
$$G(f) = -2j\int_{0}^{+\infty} g(t)\sin(2\pi f t)\,dt \,.$$

Se g(t) e' privo di simmetrie, puo' essere comunque decomposto nella somma della sua parte pari e della sua parte dispari:

$$g(t) = g_p(t) + g_d(t) \,;\quad g_p(t) = \frac{1}{2}\left[g(t) + g(-t)\right];\quad g_d(t) = \frac{1}{2}\left[g(t) - g(-t)\right].$$

Per cui da quanto detto dovra' risultare:

$$\mathbf{Re}\{G(f)\} = \mathbf{F}\,[g_p(t)] = \frac{1}{2}\mathbf{F}\,[g(t) + g(-t)]\,;$$
$$\mathbf{Im}\{G(f)\} = \mathbf{F}\,[g_d(t)] = \frac{1}{2j}\mathbf{F}\,[g(t) - g(-t)]\,.$$

9) Cambiamento si scala

$$g(a\,t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \frac{1}{|a|}G\left(\frac{f}{a}\right) ; \qquad G(b\,f) \underset{\mathbf{F}}{\overset{\mathbf{F}^{-1}}{\rightleftarrows}} \frac{1}{|b|}g\left(\frac{t}{b}\right).$$

9) Traslazione nel dominio del tempo

$$g(t - t_0) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f)e^{-j2\pi f t_0} \,.$$

10) Traslazione nel dominio della frequenza

$$G(f - f_0) \underset{\mathbf{F}}{\overset{\mathbf{F}^{-1}}{\rightleftarrows}} g(t)e^{j2\pi f_0 t} \,.$$

11) Modulazione

$$2g(t)\cos(2\pi F t + \vartheta) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f - F)\,e^{j\vartheta} + G(f + F)\,e^{-j\vartheta} \,.$$

12) Derivazione nel dominio del tempo

$$\frac{d^n}{dt^n}g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} (j2\pi f)^n\, G(f) \cdot$$

13) Derivazione nel dominio della frequenza

$$[t]^n\, g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \left(\frac{j}{2\pi}\right)^n \frac{d^n}{dt^n}G(f) \cdot$$

14) Integrazione nel dominio del tempo

$$\int_{-\infty}^{t} g(\tau)d\tau \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\,\delta(f) \cdot$$

15) Convoluzione nel dominio del tempo

$$g(t) \otimes z(t) = \int_{-\infty}^{+\infty} g(\tau)z(t-\tau)d\tau \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f)Z(f) \cdot$$

16) Convoluzione nel dominio della frequenza

$$g(t)z(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) \otimes Z(f) = \int_{-\infty}^{+\infty} G(\upsilon)Z(f-\upsilon)d\upsilon \cdot$$

17) Correlazione nel dominio del tempo

$$\int_{-\infty}^{+\infty} g(t)z^*(t-\tau)dt = \int_{-\infty}^{+\infty} g(t+\tau)z^*(t)dt \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f)Z^*(f) \cdot$$

18) Correlazione nel dominio della frequenza

$$g(t)z^*(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \int_{-\infty}^{+\infty} G(f+\upsilon)Z^*(\upsilon)d\upsilon \cdot$$

19) Relazione di Parseval

$$\int_{-\infty}^{+\infty} g(t)z^*(t)dt = \int_{-\infty}^{+\infty} G(f)Z^*(f)df \cdot$$

20) Campionamento nel dominio del tempo

$$g(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) \Rightarrow g_c(t) = \sum_{n=-\infty}^{+\infty} g(nT_c)\delta(t-nT_c) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G_c(f) = \frac{1}{T_c}\sum_{n=-\infty}^{+\infty} G\left(f - \frac{n}{T_c}\right).$$

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2 TRASFORMATE NOTEVOLI

1) Impulso ideale (impulso di Dirac)

$$g(t) = A\delta(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = A \cdot$$

2) Segnale costante

$$g(t) = A \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = A\delta(f) \cdot$$

3) Impulso rettangolare

$$g(t) = A\,\mathrm{rect}_T(t) = \begin{cases} A & \text{per } |t| \le T/2 \\ 0 & \text{per } |t| \ge T/2 \end{cases} \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = AT\mathrm{sinc}(\pi f T) \cdot$$

4) Impulso triangolare

$$g(t) = A\,\mathrm{tri}_T(t) = \begin{cases} A\left[1 - \dfrac{|t|}{T}\right] & \text{per } |t| \le T \\ 0 & \text{per } |t| \ge T \end{cases} \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = AT\mathrm{sinc}^2(\pi f T) \cdot$$

5) Segnale " sinc "

$$g(t) = A\,\mathrm{sinc}(\pi B t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = \frac{A}{B}\mathrm{rect}_B(f) \cdot$$

6) Segnale " sinc^2 "

$$g(t) = A\,\mathrm{sinc}^2(\pi B t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = \frac{A}{B}\mathrm{tri}_B(f) \cdot$$

7) Segnale esponenziale monolatero

$$g(t) = A e^{-\alpha t} u_{-1}(t) \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = \frac{A}{\alpha + j 2\pi f} \cdot$$

8) Segnale esponenziale bilatero

$$g(t) = A e^{-\alpha |t|} \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = A\frac{2\alpha}{(\alpha)^2 + (2\pi f)^2} \cdot$$

9) Segnale gradino unitario

$$g(t) = A u_{-1}(t) = \begin{cases} A & \text{per } t \ge 0 \\ 0 & \text{per } t \le 0 \end{cases} \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = \frac{A}{2}\delta(f) + \frac{A}{j 2\pi f} \cdot$$

10) Funzione " segno "

$$g(t) = A\,\mathrm{sign}(t) = \begin{cases} +A & \text{per } t \ge 0 \\ -A & \text{per } t \le 0 \end{cases} \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = \frac{A}{j\pi f} \cdot$$

11) Segnale esponenziale complesso (fasore)

$$g(t) = A e^{j 2\pi f_0 t} \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} G(f) = A\delta(f - f_0) \cdot$$

12) <u>Segnale cosinusoidale</u>

$$g(t) = A\cos(2\pi f_0 t + \vartheta) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = \frac{A}{2} e^{j\vartheta} \delta(f - f_0) + \frac{A}{2} e^{-j\vartheta} \delta(f + f_0)\,.$$

13) <u>Segnale sinusoidale</u>

$$g(t) = A\sin(2\pi f_0 t + \vartheta) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = \frac{A}{2j} e^{j\vartheta} \delta(f - f_0) - \frac{A}{2j} e^{-j\vartheta} \delta(f + f_0)\,.$$

14) <u>Treno periodico di impulsi di Dirac</u>

$$g(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} \delta\left(f - \frac{n}{T}\right).$$

15) <u>Segnale a rampa</u>

$$g(t) = A \cdot t \cdot \mathrm{rect}_T(t) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = j\frac{AT^2}{2} \frac{1}{\pi f T}\left[\cos(\pi f T) - \mathrm{sinc}(\pi f T)\right].$$

16) <u>Impulso Gaussiano</u>

$$g(t) = A e^{-\pi(t/T)^2} \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = AT\, e^{-\pi(fT)^2}\,.$$

17) <u>Impulso bipolare</u>

$$g(t) = A\,\mathrm{sign}(t)\,\mathrm{rect}_{2T}(t) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = -2jAT\mathrm{sinc}(\pi f T)\sin(\pi f T)\,.$$

18) <u>Impulso cosinusoidale</u>

$$g(t) = A\cos\left(\pi\frac{t}{T}\right)\mathrm{rect}_T(t) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = \frac{AT}{2}\left[\mathrm{sinc}(\pi f T - \pi/2) + \mathrm{sinc}(\pi f T + \pi/2)\right].$$

19) <u>Impulso modulato</u>

$$g(t) = A\,\mathrm{rect}_T(t)\cos(2\pi f_0 t) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = \frac{AT}{2}\left[\mathrm{sinc}(\pi(f - f_0)T) + \mathrm{sinc}(\pi(f + f_0)T)\right].$$

20) <u>Impulso coseno rialzato</u>

$$g(t) = A\left[1 + \cos\left(2\pi\frac{t}{T}\right)\right]\mathrm{rect}_T(t) \quad \underset{\mathbf{F}^{-1}}{\overset{\mathbf{F}}{\rightleftarrows}} \quad G(f) = AT\frac{\mathrm{sinc}(\pi f T)}{1 - (fT)^2}$$

$$G(f) = AT\mathrm{sinc}(\pi f T) + \frac{AT}{2}\mathrm{sinc}(\pi f T - \pi) + \frac{AT}{2}\mathrm{sinc}(\pi f T + \pi).$$